

# Wide binaries as a critical test of classical gravity

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**Abstract** Modified gravity scenarios where a change of regime appears at acceleration scales  $a < a_0$  have been proposed. Since for  $1M_\odot$  systems the acceleration drops below  $a_0$  at scales of around 7000 AU, a statistical survey of wide binaries with relative velocities and separations reaching  $10^4$  AU and beyond should prove useful to the above debate. We apply the proposed test to the best currently available data. Results show a constant upper limit to the relative velocities in wide binaries which is independent of separation for over three orders of magnitude, in analogy with galactic flat rotation curves in the same  $a < a_0$  acceleration regime. Our results are suggestive of a breakdown of Kepler's third law beyond  $a \approx a_0$  scales, in accordance with generic predictions of modified gravity theories designed not to require any dark matter at galactic scales and beyond.

## 1 Introduction

Over the past few years the dominant explanation for the large mass to light ratios inferred for galactic and metagalactic systems, that these are embedded within massive dark matter halos, has begun to be challenged. Direct detection of the dark matter particles, in spite of decades of extensive and dedicated searches, remains lacking. This has led some to interpret the velocity dispersion measurements of stars in the local dSph galaxies (e.g. [2, 13]), the extended and flat rotation curves of spiral galaxies (e.g. [19, 24]), the large velocity dispersions of galaxies in clusters (e.g. [21]), stellar dynamics in elliptical galaxies (e.g. [23]), the gravitational lensing due to massive galaxies (e.g. [9, 31]), and even the cosmologically inferred matter content for the universe through CMB and structure formation physics (e.g. [12, 22, 28]), not as indirect evidence for the existence of a dominant dark matter component, but as direct evidence for the failure

of the current Newtonian and general relativistic theories of gravity, in the large scale or low acceleration regimes relevant for the above situations.

Numerous alternative theories of gravity have recently appeared (e.g. TeVeS of [3], and variations; [5, 29], F(R) theories e.g. [6, 7, 27], conformal gravity theories e.g. [18]), mostly grounded on geometrical extensions to general relativity, and leading to laws of gravity which in the large scale or low acceleration regime, mimic the MODified Newtonian Dynamics (MOND) fitting formulae. Similarly, [20] have explored MOND not as a modification to Newton's second law, but as a modified gravitational force law in the Newtonian regime, finding a good agreement with observed dynamics across galactic scales without requiring dark matter. In fact, recently [4] have constructed an  $f(R)$  extension to general relativity which in the low velocity limit converges to the above approach.

Whilst classical gravity augmented by the dark matter hypothesis provides a coherent and unified interpretation from galactic to cosmological scales (with the inclusion of dark energy), the very profusion of modified gravity theories, mostly tested in very localised situations, points to the lack of any definitive theoretical contender to classical gravity. Nonetheless, a generic feature of all of the modified gravity schemes mentioned above is the appearance of an acceleration scale,  $a_0$ , above which classical gravity is recovered, and below which the dark matter mimicking regime appears. The latter feature results in a general prediction; all systems where  $a \gg a_0$  should appear as devoid of dark matter, and all systems where  $a \ll a_0$  should appear as dark matter dominated, when interpreted under classical gravity. It is interesting that no  $a \gg a_0$  system has ever been detected where dark matter needs to be invoked, in accordance with the former condition. On the other hand, the latter condition furnishes a testable prediction, in relation to the orbits of wide binaries. For test particles in orbit around a  $1M_\odot$  star, in circular orbits of radius  $s$ , the acceleration is expected to drop below  $a_0 \approx 1.2 \times 10^{-10} \text{ m/s}^2$  for  $s > 7000 \text{ AU} = 3.4 \times 10^{-2} \text{ pc}$ . The above provides a test for the dark matter/ modified theories of gravity debate; the relative velocities of components

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of binary stars with large physical separations should deviate from Kepler's third law under the latter interpretation.

More specifically, seen as an equivalent Newtonian force law, beyond  $s \approx 7000$  AU the gravitational force should gradually switch from the classical form of  $F_N = GM/s^2$  to  $F_{MG} = (GMa_0)^{1/2}/s$ , and hence the orbital velocity,  $V^2/s = F$ , should no longer decrease with separation, but settle at a constant value, dependent only on the total mass of the system through  $V = (GMa_0)^{1/4}$ . That is, under modified gravity theories, binary stars with physical separations beyond around 7000 AU should exhibit "flat rotation curves" and a "Tully–Fisher relation", as galactic systems in the same acceleration regime do.

An interesting precedent in this sense is given by the recent results of [25] and [26] who find evidence for a transition in the dynamics of stars in the outer regions of a series of Galactic globular clusters. These authors report a flattening of the velocity dispersion profile in globular clusters, outwards of the radius where accelerations fall below the  $a_0$  threshold, in accordance with generic predictions of modified gravity schemes. The interpretation under Newtonian dynamics explains the observed flattening as due to tidal heating by the Milky Way, e.g. [17], but the matter is still being debated. We also note the recent results of [16] who point out various discrepancies between standard  $\Lambda$ CDM predictions and structural and dynamical properties of the local group, and suggest solutions to these in the context of modified gravity theories.

In this paper we propose that wide binary orbits may be used to test Newtonian gravity in the low acceleration regime. We apply this test to the binaries of two very recent catalogues containing relative velocities and separations of wide binaries. The two catalogues are entirely independent in their approaches. The first one, [27] uses data from the *Hipparcos* satellite to yield a moderate number of systems (280) relatively devoid of false positives (10%), with a high average signal to noise ratio for the relative velocities of the binaries ( $\sim 2$ ). The second, [10] identifies 1,250 wide binaries from the Sloan Digital Sky Survey (SDSS) data base release 7, which, compounded with a detailed galactic stellar distribution model, results in pairs with a very low probability of chance alignment ( $< 2\%$ ), albeit with a low average signal to noise ratio in their relative velocities ( $\sim 0.5$ ).

The paper is organised as follows: Sect. 2 briefly gives the expectations for the distribution of relative velocities as a function of separation for wide binaries, under both Newtonian gravity and generically for modified theories of gravity. In Sect. 3 we show the results of applying the test to the [27] *Hipparcos* catalogue, and to the independent [10] SDSS data. Our conclusions are summarised in Sect. 4.

## 2 Expected relative velocity distributions for wide binaries

Since orbital periods for  $1M_\odot$  binaries with separations in the tens of AU range already extend into the centuries, there is no hope of testing the prediction we are interested in through direct orbital mapping. Fortunately, modern relative proper motion studies do reach binary separations upwards of  $10^4$  AU, e.g. [1, 8, 10]. The Newtonian prediction for the relative velocities of the two components of binaries having circular orbits, when plotted against the binary physical separation,  $s$ , is for a scaling of  $\Delta V \propto s^{-1/2}$ , essentially following Kepler's third law, provided the range of masses involved were narrow.

In a relative proper motion sample however, only two components of the relative velocity appear, as velocity along the line of sight to the binary leads to no proper motion. Thus, orbital projection plays a part, with systems having orbital planes along the line of sight sometimes appearing as having no relative proper motions. A further effect comes from any degree of orbital ellipticity present; it is hence clear that the trend for  $\Delta V \propto s^{-1/2}$  described above, will only provide an upper limit to the distribution of projected  $\Delta V$  vs.  $s$  expected in any real observed sample, even if only a narrow range of masses is included. One should expect a range of measured values of projected  $\Delta V$  at a fixed observed projected  $s$ , all extending below the Newtonian limit, which for equal mass binaries in circular orbits gives

$$\Delta V_N = 2 \left( \frac{GM}{s} \right)^{1/2}. \quad (1)$$

The problem is complicated further by the dynamical evolution of any population of binaries in the Galactic environment. Over time, the orbital parameters of wide binaries will evolve due to the effects of Galactic tidal forces. Also, dynamical encounters with other stars in the field will modify the range of separations and relative velocities, specially in the case of wide binaries. To first order, one would expect little evolution for binaries tighter than the tidal limit of the problem, and the eventual dissolution of wider systems.

A very detailed study of all these points has recently appeared, [15]. These authors numerically follow populations of 50,000  $1M_\odot$  binaries in the Galactic environment, accounting for the evolution of the orbital parameters of each due to the cumulative effects of the Galactic tidal field at the solar radius. Also, the effects of close and long range encounters with other stars in the field are carefully included, to yield a present day distribution of separations and relative velocities for an extensive population of wide binaries, under Newtonian gravity. Interestingly, one of the main findings is that although little evolution occurs for separations below the effective tidal radius of the problem, calculated to be of 1.7 pc, the situation for greater separations is much more complex than the simple disappearance of such pairs.

It is found that when many wide binaries cross their Jacobi radius, the two components remain fairly close by in both coordinate and velocity space, drifting in the Galactic potential along very similar orbits. This means that in any real wide binary search a number of wide pairs with separations larger than their Jacobi radii will appear. Finally, [15] obtain the RMS one-dimensional relative velocity difference,  $\Delta V_{1D}$ , projected along an arbitrary line of sight, for the entire populations of binaries dynamically evolved over 10 Gyr to today, for a distribution of initial ellipticities, as plotted against the projected separation on the sky for each pair. The expected Keplerian fall of  $\Delta V_{1D} \propto s^{-1/2}$  for separations below 1.7 pc is obtained, followed by a slight rise in  $\Delta V_{1D}$  as wide systems cross the Jacobi radius threshold.  $\Delta V_{1D}$  then settles at RMS values of  $\approx 0.1$  km/s.

These authors also explore variation in the initial (realistic) distribution of semi-major axes, and the formation history of the binaries, finding slight differences in the results, which however are quite robust to all the variations in the parameters explored, in the regime we are interested of present day projected separations larger than  $\log(s/\text{pc}) > -2$ , above very small variations of less than 0.14 in the logarithm. This represents the best currently available estimate of how relative velocities should scale with projected separations for binary stars (both bound and in the process of dissolving in the Galactic tides) under Newtonian gravity.

The testable quantitative prediction of Classical Gravity for the distribution of data in a plot of projected  $\Delta V$  vs. projected  $s$  is clear: one should find a spread of points extending below the limit defined by (1), with an RMS value for  $\Delta V_{1D}$  given by the results of [15], their Fig. 7.

The recent proliferation of modified gravity models however, implies the absence of a definitive alternative to classical gravity. Further, in many cases, the complex formulations put forth do not lend themselves to straightforward manipulations from which detailed predictions might be extracted for varied applications distinct from the particular problems under which such models are presented. We shall therefore not attempt to test any particular modified gravity theory, but shall only consider the generic predictions such theories make for “flat rotation curves” in the  $a < a_0$  regime. That is, the predictions of modified gravity schemes will only be considered qualitatively and generically, to first order, in terms of the upper envelope of observed distributions in projected  $\Delta V$  vs.  $s$  plots to appear flat. For circular orbits one expects:

$$\Delta V_{\text{MG}} = C(GMa_0)^{1/4}, \tag{2}$$

where  $C$  is a model-dependent constant expected to be of order unity. A further correction upwards due to departures from circularity, which at this point must be thought of as dependent on the details of the particular modified gravity scheme one might pick, should also be included. This correction will tend to give even larger values of  $\Delta V_{\text{MG}}$ . We

note that in the particular case of MOND, the external field effect, the fact that the overall potential of the Galaxy at the solar neighbourhood globally puts local binaries close to the  $a = a_0$  threshold, would imply only slight corrections on Newtonian predictions.

We see that all we need is a large sample of relative proper motion and binary separation measurements to test the Newtonian prediction for the RMS values of the 1 dimensional relative velocities of [15], and to contrast the  $\Delta V_N \propto s^{-1/2}$  and the  $\Delta V_{\text{MG}} = \text{cte.}$  predictions for the upper envelope of the  $\Delta V$  vs.  $s$  distributions. It is important to have a sample as free of chance alignments as possible, as the inclusion of non-physical stellar pairs would blur the test, potentially making a conclusion suspect. Also, it is desirable to limit the range of masses of the stars involved, as a spread in mass will also blur any trends expected for the upper limit of the  $\Delta V$  distributions, although not terribly so, given the small powers to which mass appears in both predictions.

To end this section we briefly recall the first order tidal limit calculation of

$$\left. \frac{dF_{\text{ext}}(R)}{dR} \right|_{R_0} \Delta r = \frac{GM_s^2}{(\Delta r)^2} \tag{3}$$

which leads to the tidal density stability condition of  $\rho_s > \bar{\rho}$  for the density of a satellite of extent  $\Delta r$  and mass  $M_s$  orbiting at a distance  $R_0$  from the centre of a spherical mass distribution  $M(R)$  having an average matter density  $\bar{\rho}$  internal to  $R_0$  resulting in a gravitational force  $F_{\text{ext}}(R)$ , under the assumption  $\Delta r \ll R$ . The equivalent calculation under the force law given by the  $a \ll a_0$  limit of  $F_{\text{MG}} = (GMa_0)^{1/2}/R$  is given by

$$\frac{(GM(R)a_0)^{1/2}}{R^2} \Delta r = \frac{(GM_s a_0)^{1/2}}{\Delta r}, \tag{4}$$

leading to

$$\rho_s > \left( \frac{\Delta r}{R} \right) \bar{\rho}, \tag{5}$$

as the equivalent of the classical tidal density criterion, as a first generic approximation under modified gravity. Since the spatial extent of wide binaries will always be much smaller than their Galactocentric radii, (5) shows that under modified gravity, to first order, wide binaries will be much more robust to tides than under Newtonian gravity. In the following section we apply the test we have identified to two recent catalogues of wide binaries which became available over the previous year, the SLoWPoKES catalogue of SDSS wide binaries by [10] and the *Hipparcos* satellite wide binaries catalogue of [27].

### 3 Observed wide binary Samples

#### 3.1 The Hipparcos wide binaries

The [27] catalogue of very wide binaries was constructed through a full Bayesian analysis of the combined *Hipparcos* data base, the new reduction of the *Hipparcos* catalogue, [30], the Tycho-2 catalogue, [14] and the *Tycho* double star catalogue, [11] mostly, amongst others. There, probable wide binaries are identified by assigning a probability above chance alignment to the stars analysed by carefully comparing to the underlying background (and its variations) in a 5 dimensional parameter space of proper motions and spatial positions. The authors have taken care to account for the distortions introduced by the spherical projection on the relative proper motion measurements,  $\Delta\mu$ . When angular separations cease to be small, small relative physical velocities between an associated pair of stars might result in large values of  $\Delta\mu$ . A correction of this effect is introduced, to keep  $\Delta\mu$  values comparable across the whole binary separation range studied.

We have taken this catalogue and kept only binaries with a probability of non-chance alignment greater than 0.9. The wide binary search criteria used by the authors requires that the proposed binary should have no near neighbours; the projected separation between the two components is thus always many times smaller than the typical interstellar separation, see [27]. We use the reported distances to the primaries, where errors are smallest, to calculate projected  $\Delta V$  and projected  $s$  from the measured  $\Delta\mu$  and  $\Delta\theta$  values reported by [27]. Although the use of *Hipparcos* measurements guarantees the best available quality in the data, we have also further pruned the catalogue to remove all binaries for which the final signal to noise ratio in the relative velocities on the plane of the sky was lower than 0.3.

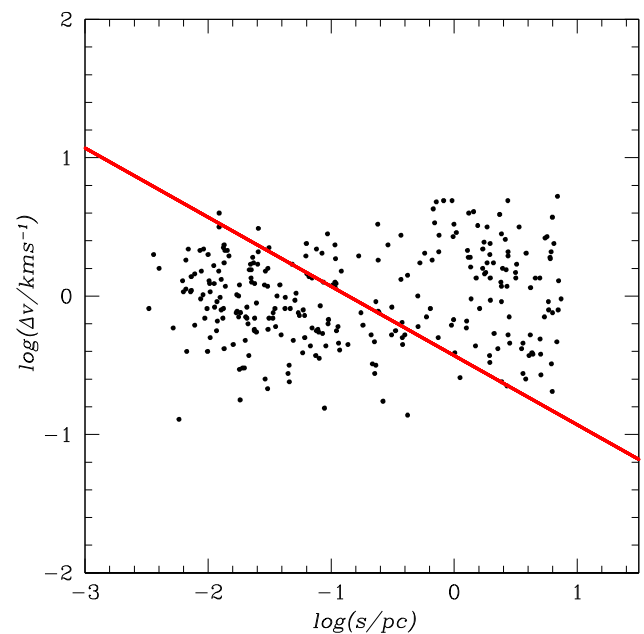
We plot in Fig. 1 a sample of 280 binaries constructed as described above, having distances to the Sun within  $6 < d < 100$  in pc. The slanted line gives the Newtonian prediction of (1) to the upper limit expected on the relative velocities shown, which appears in conflict with it, as they are defined by a neat horizontal upper limit, as generically predicted by modified gravity theories, (2). Figure 1 could then be a first direct evidence of the breakdown of classical gravity theories in the low acceleration regime of  $a < a_0$ .

The average signal to noise ratio for the data in Fig. 2 is 1.7, with an average error on  $\Delta V$  of 0.83 km/s, which considering a  $2\sigma$  factor from the top of the distribution to the real underlying upper limit for the sample, results in 3 km/s as our estimate of the actual physical upper limit in  $\Delta V$ . Comparing with (2), the factor accounting for non-circular orbits in modified gravity comes to 4.5. That this factor is significantly larger than the  $\sqrt{2}$  of Newtonian gravity is to be expected, as objects are much more tightly bound in MOND-type schemes.

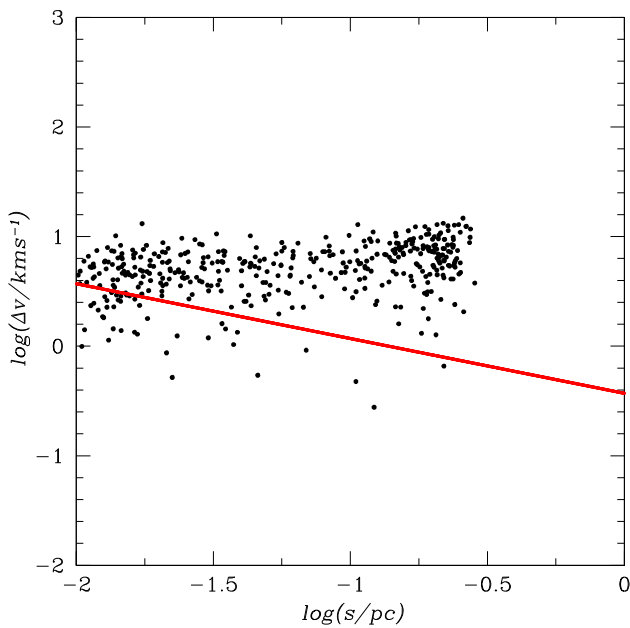
#### 3.2 The SDSS wide binaries

The Sloan low mass wide pairs catalogue (SLoWPoKES) of [10] contains a little over 1,200 wide binaries with relative proper motions for each pair, distances and angular separations. Also, extreme care was taken to include only physical binaries, with a full galactic population model used to exclude chance alignment stars using galactic coordinates and galactic velocities, resulting in an estimate of fewer than 2% of false positives. As with the *Hipparcos* sample, this last requirement yields only isolated binaries with no neighbours within many times the internal binary separation. We have also excluded all systems with white dwarfs or subdwarf primaries, where distance calibrations are somewhat uncertain. As was also done for the *Hipparcos* sample, all triple systems reported in the catalogue were completely excluded from the analysis.

Given the large range of distances to the SDSS binaries ( $46 < d/\text{pc} < 992$ ), we select only 1/3 of the sample lying within the narrow distance range ( $225 < d/\text{pc} < 338$ ), which forms the most homogeneous set in terms of the errors in  $\Delta V$ , excluding data with large errors at large distances. Again, we use the reported distances to the primaries, where errors are smallest, to calculate projected  $\Delta V$  and projected  $s$  from the measured  $\Delta\mu$ ,  $\Delta\theta$  and  $d$  values reported by [10], to plot Fig. 2. The figure shows 417 binaries with average signal to noise ratio and average errors on  $\Delta V$  of 0.5 and 11.3 km/s, respectively.



**Fig. 1** The figure shows projected relative velocities and separations for each pair of wide binaries from the [27] *Hipparcos* catalogue having a probability of being the result of chance alignment  $< 0.1$ . The average value for the signal to noise ratio for the sample shown is 1.7. The upper limit shows the flat trend expected from modified gravity theories, at odds with Kepler's third law, shown by the  $s^{-1/2}$  solid line



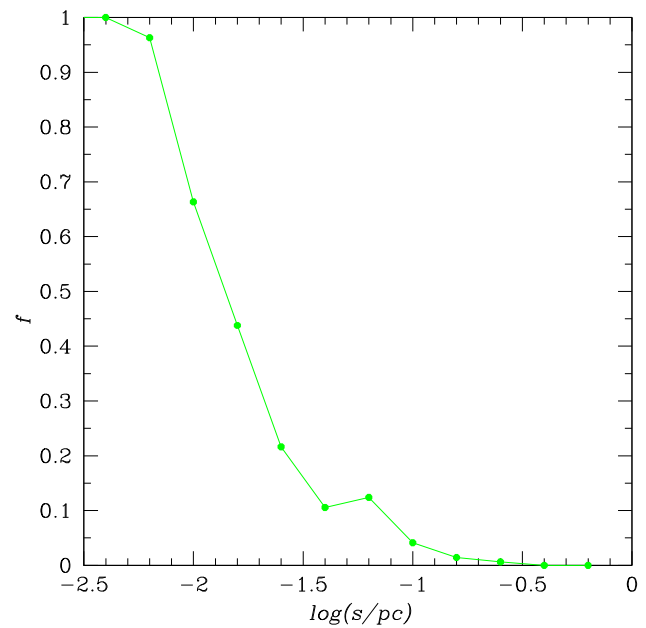
**Fig. 2** The figure shows projected relative velocities and separations for each pair of wide binaries from the [10] SDSS catalogue within the distance range ( $225 < d/\text{pc} < 338$ ). The average value for the signal to noise ratio for the sample shown is 0.5. The upper limit shows the flat trend expected from modified gravity theories, at odds with Kepler’s third law, shown by the  $s^{-1/2}$  solid line

The slanted solid line gives the Newtonian prediction of (1). It is clear that the upper envelope of the distribution of  $\Delta V$  measurements from the catalogue does not comply with Kepler’s third law. As was the case with the *Hipparcos* sample, the upper envelope of the distribution of observed measurements describes a flat line, as expected under modified gravity schemes.

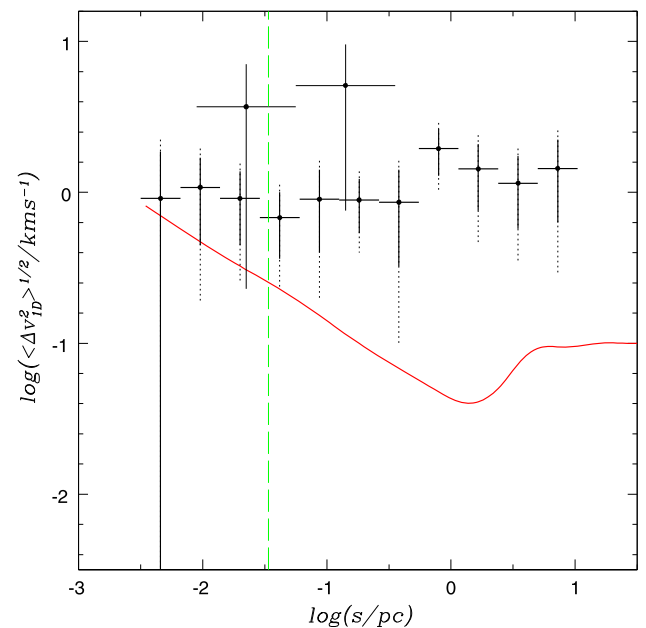
Figure 3 is a plot of the number of SLOWPoKES systems for the full distance range, with  $\Delta V$  values below the Newtonian prediction of (1), and hence consistent with it, as a fraction of the total per bin, as a function of projected binary separations. We see this fraction starts off being consistent with 1, but begins to decrease on approaching  $\log(s/\text{pc}) \approx -2$ , after which point it rapidly drops, to end up consistent with 0 on reaching separations of around  $\log(s/\text{pc}) \approx -1$ . Again, the result matches the qualitative generic expectations of modified gravity schemes, but would call for further explanations under classical gravity.

The average signal to noise values in  $\Delta V$  for the full distance range of the [10] catalogue is 0.48. The average error on  $\Delta V$  for the full SDSS sample is 12 km/s, which considering a  $2\sigma$  factor from the top of the complete distribution to the real underlying upper limit, results in the same 3 km/s as obtained for the [27] *Hipparcos* catalogue.

We end this section with Fig. 4, where we calculate the RMS value of the one-dimensional relative velocity difference for both of the samples discussed, after binning the



**Fig. 3** The figure shows the number of systems with  $\Delta V$  values below the Newtonian upper limit prediction of (1), as a fraction of the total per bin, for the SDSS data of [10], as a function of projected binary separations



**Fig. 4** The solid curve gives the RMS values for one dimensional projected relative velocities as a function of projected separations, for the detailed dynamical modelling of large populations of wide binaries evolving in the Galactic environment, taken from [15]. The same quantity for the data from the catalogues analysed is given by the points with error bars; those with narrow  $\log(s)$  intervals being from the *Hipparcos* sample of [27], and those two with wide  $\log(s)$  intervals coming from the SDSS sample of [10]

data into constant logarithmic intervals in  $s$ . This quantity is given by the points, where the error bars simply show the propagation of the errors on  $\Delta\mu$  and  $d$ , reported by the au-

thors of the catalogues. We construct  $\Delta V_{1D}$  by considering only one coordinate of the two available from the relative motion on the plane of the sky. Thus, each binary can furnish two  $\Delta V_{1D}$  measurements, which statistically should not introduce any bias. Indeed, using only  $\Delta\mu_l$  or only  $\Delta\mu_b$  or both for each binary, yields the same mean values for the points shown. The small solid error bars result from considering an enlarged sample where each binary contributes two  $\Delta V_{1D}$  measurements, while the larger dotted ones come from considering each binary only once, and do not change if we consider only  $\Delta\mu_l$  or only  $\Delta\mu_b$ . The series of small  $\log(s)$  interval data are for the *Hipparcos* catalogue of [27], while the two broader crosses show results for the [10] SDSS sample of Fig. 2. For this last case, the much larger intrinsic errors mean that to compensate through weight of numbers the low signal to noise ratio of this catalogue, imposes the loss of separation resolution through the use of only two bins, with vertical error bars which are only relevant if the sample is doubled, as described above.

The solid curve is the Newtonian prediction of the full Galactic evolutionary model of [15] for a randomly oriented population of wide binaries with a realistic distribution of eccentricities, both bound and in the process of dissolving. Note that the results of this simulation deviate from Kepler's law for  $s$  larger than the Newtonian Jacobi radius of the problem of 1.7 pc, whereas the deviation shown by the samples of binaries studied also occur at much smaller separations (see below). Even considering the large error bars, where each binary contributes only one  $\Delta V_{1D}$  value, we see eight points lying beyond  $1\sigma$ , making the probability of consistency between this prediction and the observations of less than  $(0.272)^8 = 3 \times 10^{-5}$ . The only point where this model is slightly at odds with the selection of the *Hipparcos* sample of [27], is that [15] assume  $1M_\odot$  stars for their binaries, while the typical mass of the stars in the binaries we examine is closer to  $0.5M_\odot$ . This detail would only shift the Newtonian prediction a factor of  $2^{1/2}$  further away from the measurements. We obtain an RMS value for  $\Delta V_{1D}$  compatible with a horizontal line at 1 km/s, in qualitative agreement with expectations from modified gravity schemes. The vertical dashed line marks the  $a = a_0$  threshold; we clearly see the data departing from the Newtonian prediction outwards of this line, and not before.

Under Newtonian Gravity one would need to look for an alternative dynamical dissociation and heating mechanism for the binaries we analyse which might result in relative velocities an order of magnitude above the results obtained by [15]. The *Hipparcos* catalogue has been closely studied for over a decade, and not only the values reported, but also the uncertainties in them are now well established. It is highly unlikely that these confidence intervals might have been systematically underestimated by the community by a factor of 3, as would be required for the data in Fig. 4 to be consistent with the Newtonian prediction of [15].

The two SDSS points are clearly consistent with the *Hipparcos* measurements, but given the much larger error bars, they are also marginally consistent with the Newtonian prediction at a  $2\sigma$  level. A fuller discussion of the robustness of our results to the various sample selection effects and errors involved appears in the appendix.

The consistency of the results obtained from both catalogues provides a check of the physical reality of the trends presented. The two completely independent, very carefully constructed catalogues, each using different sets of selection criteria, each perhaps subject to its own independent systematics, are consistent with the same result, a constant horizontal upper envelope for the distribution of relative velocities on the plane of the sky at an intrinsic value of  $3 \text{ km/s} \pm 1 \text{ km/s}$ , extending over three orders of magnitude in  $s$ , with a constant RMS  $\Delta V_{1D}$  value consistent with  $1 \text{ km/s} \pm 0.5 \text{ km/s}$ . This supports the interpretation of the effect detected as the generic prediction of modified gravity theories.

Given the relevance of the subject matter discussed, it would be highly desirable to obtain an independent confirmation, ideally using a purposely designed sample including cuts at a variety of stellar masses, in order to test the scaling of the fourth power of the dynamical velocities with mass expected under modified gravity schemes. To include also radial velocity measurements would require sampling over many epochs, as the only way to allow for the effects of nearby companions in these frequently hierarchical systems.

## 4 Conclusions

We have identified a critical test in the classical gravity/modified gravity debate, using the relative velocities of wide binaries with separations in excess of 7000 AU, as these occupy the  $a < a_0$  regime characteristic of modified gravity models. We present a first application of this critical test using the best currently available data; a large sample of wide binaries from the SDSS with low signal to noise on the relative velocities, and a smaller *Hipparcos* satellite sample with signal to noise  $\sim 2$  on the relative velocities of the binaries sampled.

Results show constant relative RMS velocities for the binary stars in question, irrespective of their separation, in the  $a < a_0$  regime sampled. This is quantitatively inconsistent with detailed predictions of Newtonian dynamical models for large populations of binaries evolving in the local galactic environment.

Our results are qualitatively in accordance with generic modified gravity models constructed to explain galactic dynamics in the absence of dark matter, where one expects constant relative velocities for binary stars, irrespective of their separation, in the  $a < a_0$  regime sampled.

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**Appendix: Calculation of confidence intervals**

We begin this section with a discussion of various analyses performed to test for the possibility that our results could have been driven by potential systematics and selection effects in the catalogues.

We first test for the option that the results of Figs. 1 and 2 were distorted from the Newtonian prediction by errors which correlate with the separation of the binaries, increasing as the separation increases, to yield the trends obtained. For the *Hipparcos* sample we ranked the binaries by separation,  $s$ , and calculated the average errors on the resulting  $\Delta V$  in each the tightest third, middle third and widest third of the binaries, yielding values 0.8 (0.4), 0.7 (0.11) and 1.0 (0.4) respectively, in km/s. The numbers in parentheses giving the dispersion of the distributions of errors in each of the three thirds of the sample. It can be seen that there is no increase either in the average values of the errors in  $\Delta V$ , or in the width of the distributions of errors, with increasing binary separations. For the SDSS sample the average errors in  $\Delta V$  show only a very slight increase of a factor of 1.3 over the entire range probed. Thus, for both samples, the trends of Figs. 1, 2 and 4 cannot be explained as arising from the Newtonian prediction and an increase in the errors with binary separation. In essence, the data presented are inconsistent with the Newtonian prediction not because of differences in the details of the trends, but because the former presents multiple real detections at a level of 3 km/s  $\pm$  1 km/s, while the latter is smaller by an order of magnitude. The resulting RMS values for the various data points are of  $\approx$ 1 km/s  $\pm$  0.5 km/s, while the Newtonian prediction lies below the detection by about a factor of 10.

We next check against systematics in the catalogues, which would preferentially arise in the lower signal to noise points, or alternatively, in the most distant ones. For the *Hipparcos* sample, we repeat the experiment including only points with a signal to noise ratio on  $\Delta V > 1$ , and then including only the binaries nearest to earth with  $d < 50$  pc. These two sub-samples have 186 and 130 binaries, with average signal to noise ratios on  $\Delta V$  above the value of 1.7 of Fig. 1, of 2 and 2.2 respectively, and yield results indistinguishable from Figs. 1 and 4. Also, we checked explicitly the results of the SDSS sample for any systematics with distance to the binaries analysed, and found the flat upper envelope to be robust to the choice of distance range taken.

Although no specific cut in resulting  $\Delta V$  or even on measured  $\Delta\mu$  was built into the *Hipparcos* catalogue, the SDSS

one includes the selection cut that the signal to noise ratio on  $\Delta\mu$  should be  $< 2$ . We repeat the SDSS analysis tracing the 15% of the points closest to this cut, and find that these, the ones which just made the cut, do not define the flat upper envelope on  $\Delta V$ . For both the *Hipparcos* and the SDSS samples, any observational bias/truncation would appear in  $(\Delta\mu, \Delta\theta)$  space, not in the  $(\Delta V, s)$  one.

We end with a detailed description of the calculation of the points in Fig. 4 and their error bars, directly from the data published in the catalogues used. Whenever  $z = f(x)$ , the error in  $z$  is given by  $\delta z = \left| \frac{df}{dx} \right| \delta x$ , where  $\delta x$  is the error in  $x$ . If  $z$  is a function of several variables  $z = f(x_1, x_2, \dots)$ , the absolute error, which we take, is  $\delta z = \sum_i \left| \frac{\partial f}{\partial x_i} \right|_{x_i} \delta x_i$  which is always greater than the standard deviation  $\sigma_z = \sqrt{\sum_i \left( \frac{\partial f}{\partial x_i} \delta x_i \right)^2}$ .

Following these rules we estimate the absolute error in each  $\Delta V$  measurement, and in the RMS velocity for the samples studied. For the *Hipparcos* sample we have taken columns 11, 12, 13 and 14 of the online catalogue of [27], which contains the values of the relative motions of each binary and their errors, as well as columns 15 and 16, which contain the distance to the binary,  $d$ , and its corresponding error,  $\delta d$ .

For all calculations, we considered the distance to the system as the distance to the primary. For each binary we calculated the projected relative velocity  $\Delta v$  in km/s and its error as

$$\Delta v = 4.74 \times 10^{-3} \Delta\mu d, \tag{A.1}$$

$$\delta(\Delta v) = 4.74 \times 10^{-3} (\delta(\Delta\mu)d + \Delta\mu\delta d), \tag{A.2}$$

hence, on  $(\Delta v)^2$  the error is  $\delta(\Delta v)^2 = 2\Delta v\delta(\Delta v)$ . The RMS velocity is now

$$\Delta v_{\text{RMS}} = \sqrt{\frac{1}{n} \sum_{i=1}^n (\Delta v)^2} = \sqrt{\langle (\Delta v)^2 \rangle}. \tag{A.3}$$

We then binned the data into constant logarithmic intervals in  $s$  with a width of 0.32 and calculated  $\Delta v_{\text{RMS}}$  for each bin,  $n$  the number of binary systems that fall into each bin. The error in  $\Delta v_{\text{RMS}}$  for each bin is now:

$$\delta(\Delta v_{\text{RMS}}) = \frac{1}{2} \frac{\langle \delta(\Delta v)^2 \rangle}{\sqrt{\langle (\Delta v)^2 \rangle}}. \tag{A.4}$$

To compare with the prediction of the RMS values for the one dimensional projected relative velocities of the evolutionary model of [15] we construct  $\Delta v_{\text{RMS}}$  by considering only one coordinate. For the case of the SDSS sample of [10], we have calculated the relative proper motion for each coordinate  $\Delta\mu_\alpha = \mu_{\alpha_A} - \mu_{\alpha_B}$  and  $\Delta\mu_\delta = \mu_{\delta_A} - \mu_{\delta_B}$  taking columns 18 to 25 of their online catalogue, which contain the proper motions and their errors in equatorial coordinates for each component of the binary system, to estimate the

projected relative velocity we have used column 26 containing the distance to the primary, in this case we consider an error of 15% in the distance and we have followed the same path as in the previous case for calculating the RMS relative velocities and their errors.

## References

1. C. Allen, A. Poveda, M.A. Herrera, *Astron. Astrophys.* **356**, 529 (2000)
2. G.W. Angus, *Mon. Not. R. Astron. Soc.* **387**, 1481 (2008)
3. J. Bekenstein, *Phys. Rev. D* **70**, 3509 (2004)
4. T. Bernal, S. Capozziello, J.C. Hidalgo, S. Mendoza, *Eur. Phys. J. C* **71**, 1794 (2011)
5. J.P. Bruneton, G. Esposito-Farese, *Phys. Rev. D* **76**, 124012 (2007)
6. S. Capozziello, M. De Laurentis, *Phys. Rep.* **509**, 167 (2011)
7. S. Capozziello, V.F. Cardone, A. Troisi, *Mon. Not. R. Astron. Soc.* **375**, 1423 (2007)
8. J. Chaname, A. Gould, *Astrophys. J.* **601**, 289 (2004)
9. M.C. Chiu, C.M. Ko, Y. Tian, H.S. Zhao, *Phys. Rev. D* **83**, 06352 (2011)
10. S. Dhital, A.A. West, K.G. Stassun, J.J. Bochanski, *Astron. J.* **139**, 2566 (2010)
11. C. Fabricius, E. Høg, V.V. Makarov, B.D. Mason, G.L. Wycoff, S.E. Urban, *Astron. Astrophys.* **384**, 180 (2002)
12. A. Halle, H.S. Zhao, B. Li, *Astrophys. J. Suppl. Ser.* **177**, 1 (2008)
13. X. Hernandez, S. Mendoza, T. Suarez, T. Bernal, *Astron. Astrophys.* **514**, A101 (2010)
14. E. Høg et al., *Astron. Astrophys.* **357**, 367 (2000)
15. Y.F. Jiang, S. Tremaine, *Mon. Not. R. Astron. Soc.* **401**, 977 (2010)
16. P. Kroupa et al., *Astron. Astrophys.* **523**, 32 (2010)
17. R. Lane et al., *Mon. Not. R. Astron. Soc.* **406**, 2732 (2010)
18. P.D. Mannheim, D. Kazanas, *Astrophys. J.* **342**, 635 (1989)
19. S.S. McGaugh, *Phys. Rev. Lett.* **106**, 121303 (2011)
20. S. Mendoza, X. Hernandez, J.C. Hidalgo, T. Bernal, *Mon. Not. R. Astron. Soc.* **411**, 226 (2011)
21. M. Milgrom, R.H. Sanders, *Astrophys. J.* **678**, 131 (2008)
22. M. Milgrom, *Phys. Rev. D* **82**, 043523 (2010)
23. N.R. Napolitano, S. Capozziello, A.J. Romanowsky, M. Capaccioli, C. Tortora, *Astrophys. J.* (2012, in press). [arXiv:1201.3363](https://arxiv.org/abs/1201.3363)
24. R.H. Sanders, S.S. McGaugh, *Annu. Rev. Astron. Astrophys.* **40**, 263 (2002)
25. R. Scarpa, G. Marconi, R. Gimuzzi, G. Carraro, *Astron. Astrophys.* **462**, L9 (2007)
26. R. Scarpa, G. Marconi, G. Carraro, R. Falomo, S. Villanova, *Astron. Astrophys.* **525**, A148 (2011)
27. E.J. Shaya, R.P. Olling, *Astrophys. J. Suppl. Ser.* **192**, 2 (2011)
28. C. Skordis, D.F. Mota, P.G. Ferreira, C. Boehm, *Phys. Rev. Lett.* **96**, 011301 (2006)
29. Y. Sobouti, *Astron. Astrophys.* **464**, 921 (2007)
30. F. van Leeuwen, *Astron. Astrophys.* **474**, 653 (2007)
31. H.S. Zhao, B. Famaey, *Phys. Rev. D* **81**, 087304 (2010)