Matter, Energy, Space and Time
Winter 2022, Term Paper
Robert Augusto Riva

On the Precession of the Equinoxes

Eppur si muove[1]

The Copernican model of precession is in crisis.[2] The phenomenon known as the precession of the equinoxes was first observed no later than the second century B.C. by Hipparchus, though it was not until Copernicus, nearly seventeen centuries later, that a mechanism was posited to explain that phenomenon.[3] For Copernicus, explaining the precessional phenomenon was a necessary component of his larger heliocentric model of the Solar System. Centuries of advances in that larger model, in the theoretical underpinnings on which that model is based, and in the observational tools that helped guide and confirm those advances, have led to ever greater observational accuracy and explanatory power by the model overall. Those same advances, however, have revealed discrepancies between the model's explanation of the precessional phenomenon, and the theoretical and observational underpinnings to which that explanation is ultimately accountable. Indeed, not only has a lasting solution to those discrepancies proven elusive, but contemporary attempts to refine the precessional aspects of the model have tended instead to underscore the discrepancies, pushing the Copernican model of precession to a crisis point, and perhaps even edging it towards the verge of a Kuhnian paradigm shift. This paper will attempt, in a non-technical manner, to describe the precession of the equinoxes and its current theoretical and observational underpinnings, to examine certain key difficulties in the current model of precession, and to summarize briefly an alternative model which, by introducing a purportedly missing observational reference frame, claims to improve on the current model in a way that saves both theory and observation from the current model's precessional shortcomings.

The advent of the Copernican model was, at its core, a fundamental shift in observational reference frames, from one inertial frame[4] to another. That shift in reference frames liberated Earth from its static position at the center of the Ptolemaic universe – around which moved the Sun, the other planets and the distant stars – and placed Earth and the other planets in motion around a now

---

[1] "And yet it moves," words purportedly uttered by Galileo after his trial for heresy, in which he was forced to recant the heliocentric model of the Solar System. See Livio's wonderful article on these famous words.

[2] See, for example, *IAU 2006 Resolution B1*, which noted "the need for a precession theory consistent with dynamical theory," and *IAU 2011 Resolution B2*, which noted that "the current Earth rotation theories … are unable to model and predict Earth orientation parameters (EOP) with an accuracy close to the current stringent requirements, … in spite of the improved accuracy and precision of the individual and combined solutions derived from single or multiple techniques", and resolving that "new models should be closer to the dynamically time-varying, actual Earth, and adaptable as much as possible to future updating of the reference frames and standards".

[3] Close, p. 450, and Sandeman, p. 32.

[4] An inertial reference frame is, for purposes of this paper, a coordinate system that is "at rest" with respect to the "fixed" stars. It was just such an inertial reference frame, for example, that Newton had in mind when he described space as absolute and fixed. One of Einstein's great insights was to relativize observational reference frames in a new and rigorous way. See Weinberg pp. 233-234 and 250-251 for a discussion of inertial vs relativistic reference frames.

static Sun looking out at a likewise fixed background of distant stars.[5] The Copernican model was able to explain the observed motions of the heavens in a new way, in part by positing three key terrestrial motions: (i) *axial rotation*, the diurnal counterclockwise[6] spin of Earth around its tilted axis, (ii) *orbital revolution*, Earth's annual, counterclockwise trek around the Sun, and (iii) *axial precession*, what Copernicus referred to as libration, the slow, circular, clockwise reorientation of Earth's axis around itself.[7] The first two motions together with the corresponding orbits of Earth's planetary peers around the Sun, for example, allowed Copernicus to explain the movements of the other planets much more simply, if initially also less accurately, than the geostatic Ptolemaic model which the Copernican model ultimately supplanted. The inclusion of the third motion, Earth's axial precession, was a stroke of genius that allowed Copernicus to halt the motion of all stars, not just that of the Sun, while at the same time explaining the slow, simultaneous and counterclockwise apparent motion of the distant stars over thousands of years: the precession of the equinoxes.

In order to understand the precession of the equinoxes, one must first understand what is meant by the "equinox." Earth's equator, and therefore its axis of rotation, is tilted at approximately 23.5° relative to the plane of the ecliptic. The plane of the ecliptic is an imaginary plane that extends from the center of the Sun through an ellipse described by Earth's orbit around the Sun. In other words, it is Earth's orbital plane around the Sun. Viewed from Earth, however, the ecliptic instead appears to describe the path of the Sun though the heavens during the course of the year, a path that can be plotted by observing the ever-changing location of the Sun in the sky at the same time each day. Indeed, the plane of the ecliptic is so called because it is only when the path of the Moon crosses that plane that lunar and solar eclipses occur.[8] Because the other planets of the Solar System also move around the Sun along roughly the same orbital plane as that of Earth, each planet

---

[5] The current understanding of the Sun and the distant stars, of course, is that they all move relative to one another: the Sun and the other stars in our galaxy move around the center of the Milky Way, stars in other galaxies move around their galactic centers, and galaxies and even galaxy clusters move relative to each other. Although the Solar System is in motion around our galactic center, for example, in an orbit of approximately 240,000,000 years by some estimates, that motion is thought to account for only a small portion of the precessional phenomenon; the other sidereal and galactic motions account for an even smaller portion. For purposes of this paper, then, one can simply abstract out these other motions, and essentially maintain Copernicus' original *in medio vero omnium residet Sol* (i.e., "truly in the middle of all resides the Sun"), in describing the current model of precession.

[6] What is meant by "clockwise" and "counterclockwise" here and in the rest of this paper is the relative direction of motion when viewed from a vantage point above Earth's orbital plane or, for perhaps greater clarity, above the terrestrial North Pole. The key aspect of the three motions of Earth is that while the first two motions, axial rotation and orbital revolution, are *counterclockwise*, axial precession is oppositely directed; its movement is *clockwise*. As discussed in more detail below, it is by this contrary motion of precession that the current model explains both (i) the difference between the sidereal year and the shorter equinoctial year, and (ii) the slow, counterclockwise apparent movement of the distant stars, i.e., the precessional phenomenon.

[7] Blair, p. 51.

[8] The plane of the Moon's orbit around Earth is offset approximately 5° from the plane of the ecliptic. This offset introduces an additional layer of complexity into the discussion of precession, but does not change the underlying principles. Accordingly, this paper will disregard that 5° offset and will focus the relevant portion of the discussion on the plane of the ecliptic only, as if the Moon orbited Earth along the ecliptic plane.

likewise appears, when viewed from Earth, to move more or less along the plane of the ecliptic. Moreover, while stars and constellations are scattered throughout the celestial sphere, there are twelve constellations in particular that lie along the plane of the ecliptic. These are the twelve zodiacal constellations: Aries, Pisces, Aquarius and the rest.

Within the ecliptic framework, the orientation of Earth's axis remains almost[9] perfectly constant relative to the distant stars, which means that, as a result of Earth's orbit around the Sun, Earth's axis finds itself throughout the year at continuously different angles relative to the Sun. Put another way, as Earth orbits the Sun, the North Pole of Earth's axis currently points most directly towards the Sun on a particular day in June, i.e., on the summer solstice, and most directly away from the Sun on a particular day in December, the winter solstice.[10] By definition, an equinox occurs when Earth's axis is perpendicular to a line between Earth and the Sun. That is, if one imagines a straight line from the center of the Sun to a plane along which Earth's axis lies, the equinox is the precise moment during the course of the year when that line is perpendicular to the plane of Earth's axis. That moment is fleeting, but it recurs twice during the course of each year, once in March and again in September, marking the first day of spring and the first day of autumn – the vernal and autumnal equinoxes, respectively.

Looking due east at dawn on a cloudless morning of the vernal equinox in any year during the past two millennia, one would see the Sun rising against the background of the constellation Pisces.[11] As Earth sweeps along its orbit around the Sun during the course of the year, the background constellation that lies directly behind the Sun along the plane of the ecliptic also appears to change. In February, the constellation through which the Sun rises is Aquarius; in April, it is Aries, and so on. Because Earth orbits counterclockwise around the Sun, this motion makes the distant stars, including the twelve zodiacal constellations, appear from Earth to be rotating clockwise, and every year this sidereal circuit of the Sun against the background of the distant stars completes its apparent path and begins again in an endless cycle that is both predictable and precise.

Just, not perfectly precise.

In the second century B.C., Hipparchus was comparing his observational measurements of various distant stars with those of Timocharis and Aristyllus, who made their measurements more

[9] As discussed below, it is in this "almost" that the precessional phenomenon lies.

[10] It is beyond the scope of this paper, and ultimately irrelevant to the Copernican explanation of precession, to discuss the applicable Milanković cycles that impact this orientation.

[11] This indeed is the reason why those born in the month of March are currently considered "Pisces." Moreover, because on the vernal equinox the Sun currently rises with the constellation Pisces behind it, the current "Age" as the Ancient Greeks would call it is referred to as the Age of Pisces. Because of precession, however, the apparent shifting of the equinoxes means that in a century or two, the Sun will instead rise on the vernal equinox with the constellation Aquarius behind it, and the Age of Aquarius will supersede that of Pisces, just as a little more than 2,000 years ago the Age of Pisces superseded that of Aries.

than a century and half before Hipparchus.[12] Hipparchus discovered that the star Spica Virginis, in the Virgo constellation, had been located 8° off the autumnal equinox in the time of Timocharis and Aristyllus, but was only 6° off the autumnal equinox in Hipparchus' time.[13] Hipparchus referred to this as a μετάπτωσις, or shifting, of the stars relative to the equinoctial points.[14] In Hipparchus' time, this μετάπτωσις resulted in Spica Virginis preceding its expected location relative to the autumnal equinox by 2° compared to its location at the autumnal equinox in the time of Timocharis and Aristyllus. This apparent shifting is what eventually became known as precession of the equinoxes. The slowness of this μετάπτωσις is indicated by Hipparchus's measurement of it: hardly 2° in more than century and a half. Indeed, the precise rate of μετάπτωσις – of the precessional phenomenon – is just over 1° every 72 years, meaning it would take nearly 26,000 years at that constant rate for Spica Virginis and the other distant stars to complete one full apparent precession around the equinoxes: a rate of slightly more than 50 arcseconds per year.[15] While there is good reason to think that Hipparchus "did not believe in the *actual* retrogradation of the equinoctial points,"[16] nonetheless Hipparchus did not posit any mechanism to explain his observed μετάπτωσις. That mechanism awaited Copernicus.

Copernicus attributed precession of the equinoxes to the axial precession of Earth. According to Copernicus, as Earth orbits the Sun, Earth's axis does not stay perfectly fixed relative to the distant stars. Instead, it moves very slowly in a clockwise direction relative to the distant stars. Because Earth is orbiting counterclockwise around the Sun, this clockwise "libration" of Earth's axis means that the precise moment of perpendicularity between the plane of Earth's axis and the line between it and the Sun occurs approximately 50 arcseconds *before* Earth completes a full 360° orbit around the Sun (i.e., at approximately 359 degrees, 59 arcminutes, 10 arcseconds, or 359° 59' 10" for short, into that orbit). 50 arcseconds of Earth's orbit translates into roughly 36,000 km of Earth's 940,000,000 km trek around the Sun. It takes Earth just over twenty minutes to traverse those 36,000 km, which means that an equinoctial year – the time from vernal equinox to vernal equinox (or, for Hipparchus, autumnal equinox to autumnal equinox) – is approximately twenty minutes shorter than a sidereal year, the time it takes Earth to complete a full 360° orbit around the Sun as measured by reference to the distant stars rather than the equinoxes. As a result of precession, the annual clockwise movement of the Sun through the zodiacal constellations along the plane of the ecliptic, as viewed from Earth, likewise falls approximately 50 arcseconds short each year compared to the previous year. The cumulative effect of this is that the zodiacal

---

[12] Close, p. 450.

[13] Close, p. 456, and Sandeman, pp. 31-32.

[14] In Ancient Greek, μετάπτωσις more precisely means a fleeing elsewhere, or a seeking refuge elsewhere.

[15] There are 360° of arc in a circle. Each degree is subdivided into 60 arcminutes, and each arcminute itself is subdivided into 60 arcseconds. An arcsecond is therefore 1/3,600 of a degree (or 1/1,296,000 of a circle), which means 50 arcseconds is 1/25,920 of a circle. In other words, at a constant rate of 50 arcseconds per year, it would take 25,920 years to complete one 360° rotation. (The exact period is currently estimated to be slightly shorter than this, at 25,772 years, translating into a precessional rate of approximately 50.3 arcseconds per year.)

[16] See Close generally, and in particular his conclusion at p. 456.

constellations, indeed virtually all the distant stars, appear to be shifting *en masse* counterclockwise, when viewed from Earth, at a rate of approximately 1° every 72 years, imperceptibly slow within the single lifetime of an unaided observer.

The Copernican model's description of axial precession was eventually given theoretical grounding some 150 year later by Newton, who hypothesized the motion as being the result of the combined gravitational effects of the Sun and the Moon on the oblate Earth.[17] Newton's gravitational grounding of axial precession formed, and still lies at, the core of the current lunisolar theory of axial precession. Earth is not a perfectly rigid body; rather, due to Earth's internal geophysical properties, its mass is only semi-rigid, and therefore partly elastic. As a result of the planet's axial rotation, this elasticity allows Earth's surface to deform as the planet spins, manifesting in a roughly symmetric bulge along Earth's equator, perpendicular to its spin axis. In other words, Earth is not a perfect sphere, but is instead an oblate spheroid whose equatorial radius (about 6,384 km) is greater than its polar radius (about 6,353 km). Nonetheless, one can imagine a perfect sphere lying within the interior of Earth, a sphere with a radius equal to the polar radius. That inner sphere is roughly tangential to Earth's surface at the poles, but is about 0.3% smaller than Earth's surface at the equator, a difference that is sufficient under the current lunisolar theory to cause Earth's axis to precess as a result of gravity-induced torque acting on the rotating Earth.

In Newtonian terms, every "particle" of Earth experiences a gravitational attraction between Earth and all other bodies in the Solar System.[18] Of overwhelmingly primary importance, however, is the gravitational attraction Earth experiences between itself and the Sun (in relative terms, because of the Sun's mass) and between itself and the Moon (in relative terms, because of the Moon's proximity). Because Earth is of finite size instead of being an infinitesimal point mass, not all particles of Earth are the same distance from the Sun or the Moon. If one considers any particle in the imaginary interior spherical portion of Earth mentioned above, the symmetry of that spherical portion means that there will be a similar particle at the same distance on the opposite side of the sphere along the same plane. The tidal forces on those two particles will be of equal magnitude and act in opposite directions along the same line, resulting in zero net torque for each such pair of particles, and zero net torque for the sphere overall.[19] Thus, if Earth were a perfect sphere, or if its equator were not tilted with respect to the plane of the ecliptic, it would experience no additional net torque on the angular momentum of its axial spin: Earth's axis would simply not precess.

[17] Dobson p. 345, and Robinson p. 57.

[18] See Dobson, pp. 347-349, from which this paragraph and the next are largely drawn, for a more detailed analysis of Newton's theoretical explanation of axial precession.

[19] Tidal force is a gravitational effect resulting from the gravitational force exerted by one mass on the extended body of another which, because of the extended nature of the body and the fact that the force of gravity is inversely proportional to the distance between the two masses, is not constant across the parts of the extended body. Torque is a measure of a force that can cause an object to rotate around its axis, generating angular momentum. Of relevance here, torque is a vector quantity whose direction depends on the direction of the force on that axis of rotation.

Earth, however, is an oblate spheroid that is tilted with respect to the plane of the ecliptic. As a result, particles along Earth's equatorial region – those lying within the 0.3% gap between Earth's exterior and the outer surface of that imaginary interior sphere – have no corresponding particles on the opposite side of the plane of the ecliptic. The sum of the resulting tidal forces produces a net torque tending to turn the plane of Earth's equator into the plane of the ecliptic. This net torque does not change Earth's angular momentum, or the rate of axial rotation generating that angular momentum. Rather, it adds a new component of angular momentum in the direction in which it acts, causing Earth's axis of rotation itself to reorient steadily in the direction of that new component, while maintaining Earth's 23.5° tilt to the ecliptic plane, a motion that traces out a circular cone, or precesses, in a clockwise direction. As Dobson points out of Newton's hypothesis:

> "The reason why a torque … causes this rotation about the normal to the ecliptic is not at all intuitively obvious and, at first sight, rather surprising. Its complete explanation requires ideas of rigid body kinematics and dynamics and, specifically, the ideas of moment of inertia and angular momentum. A study of Newton's treatment shows that he has neither these ideas, nor anything equivalent to them, and that he does not, from a modern point of view, give a convincing explanation of the phenomenon."[20]

In other words, Newton may have recognized the tidal force torque as the cause of axial precession, and even conceived that a quantitative explanation of it was possible, but that quantitative explanation awaited the development of ideas for dealing with the motion of extended rigid bodies. Such a discussion is beyond the scope of this paper, and it must suffice to reiterate as noted above that Newton's gravitational hypothesis grounding axial precession formed, and still lies at, the core of the current lunisolar theory of axial precession.[21]

By the end of the 19th Century, theoretical advances in rigid body kinematics and dynamics had filled in enough of what was missing in Newton to permit Simon Newcomb to calculate axial precession at 5,034.62 arcseconds per century, or just over 50.3 arcseconds per year.[22] Newcomb refined the calculations of his precessional equation over the next few years and, with succeeding advances in the observational tools by which the precessional phenomenon is measured, those calculations have been refined even further. According to Hilton et al., axial precession is "most accurately observed during Very Long Baseline Interferometer (VLBI) observations."[23] During VLBI, a radio signal from an extremely distant but powerful astronomical source, typically a

---

[20] Dobson, p. 349.

[21] Indeed, it should be noted that the name *lunisolar* is no longer favored, given that, as noted below, axial precession equations now take into account the gravitational contribution of the other planets and other Solar System bodies. See Hilton et al, pp. 352-353. Nonetheless this paper will continue to refer to the current theory underlying axial precession as the lunisolar theory.

[22] Newcomb, p. 187.

[23] Hilton et al., p. 353. See also Gatano et al., pp. 2-4.

remote quasar, is received by several radio-telescopes around Earth. This effectively turns the array of radio-telescopes into one enormous radio-telescope making exceptionally precise measurements of slight changes in the motion of Earth relative to those distant objects.

Those increasingly accurate observations have led to increased complexity in Newcomb's original equation. Hilton et al. set forth one such recent precessional equation, a complex polynomial expression where the accumulated precession p_A = 5,028.796195 arcseconds per century + 1.1054348 arcseconds per century squared + 0.00007964 arcseconds per century cubed – 0.000023857 arcseconds per century to the fourth, etc.[24] The rate of precession is the first derivative of this equation, or 5,028.796195 + 2.2108696 arcseconds per century, etc. Although the equation yields accurate predictions for small time periods, it is of limited predictive value over longer time periods.[25] A quick inspection of the exponents shows that any prediction for too many centuries into the future will yield a precessional figure that is so large as to be, at best, inconsistent with what lunisolar theory could possibly support.

More problematic yet is the underlying observation which the polynomial expression was intended to describe. Based on VLBI measurements, the rate of the precession is not constant; rather, it is increasing over time. The importance of this observation cannot be overstated. The Moon is slowly receding from Earth, and Earth is slowly receding from the Sun. Because the gravitational attraction between two bodies is inversely proportional to the square of the distance between them, any increase in the distance between the Moon and Earth, or between Earth and the Sun, should lessen the tidal force torque on Earth's spin angular momentum, not increase it. Current axial precession equations do indeed take into account the gravitational contribution of the other planets and other Solar System bodies on axial precession, but the greatest gravitational contribution to axial precession of Earth remains by far that of the Sun and the Moon. An accelerating precessional rate here would seem to contradict directly Newton's law of universal gravitation.

Another compelling piece of evidence, based on lunar observations, calls into question whether Earth's axis is precessing at all beyond, say, mere nutation and Chandler wobble.[26] If Earth is indeed undergoing axial precession, its effect should manifest not only with respect to the apparent motion of distant stars, but also with respect to the apparent motion of nearby objects such are the Moon. Lunar data from the Astronomical Almanac contradict this, however, suggesting instead that if Earth's axis is precessing, it is not doing so with respect to the Moon and indeed is perhaps doing so only with respect to the distant stars. To understand this paradox, one

[24] Hilton et al., p. 356.

[25] Hilton et al., p. 354.

[26] Nutation is a small circular harmonic in Earth's axis over an 18.6 year period, a motion attributed to the Moon's orbit around the Earth-Moon barycenter. Chandler wobble is an even smaller harmonic over a 1.2 year period, attributed to pressure fluctuations in the oceans and the atmosphere. Of critical importance is that each motion is well established as being independent of axial precession generally and, according to the International Astronomical Union, does not suffer from the modeling and observational difficulties of axial precession.

must look to the different measurements of the lunar month. There are three primary measures of the lunar month. The synodic month, which is the length of time from one new moon to the next when viewed from the same point on Earth, is slightly more than 29.5 days. The lunar sidereal month, which is the length of time for the Moon to realign with a distant star when viewed from the same point on Earth, is just under 27.3 days. The difference between the two measurements is due to the fact that as the Moon is orbiting Earth, Earth is orbiting the Sun, and a new moon occurs only when the Moon lies directly along a straight line between Earth and the Sun. Earth's orbit around the Sun requires the Moon to orbit Earth an additional 2.2 days for that alignment to be achieved with respect to each succeeding lunar orbit. The third measure of a lunar month is the lunar tropical month, which is essentially a lunar sidereal month adjusted for the effects of Earth's axial precession, analogous to the 50 arcsecond per year difference between an Earth equinoctial year and an Earth sidereal year. A lunar tropical month is only a few seconds shorter than a lunar sidereal month.

Whether or not observations of the Moon show the effects of Earth's axial precession, the Moon's orbit itself is essentially independent of any Earth wobble. It is therefore possible to compare the difference between a synodic month and a lunar tropical month in each of an Earth equinoctial year and an Earth sidereal year. The ratio of lunar tropical month to synodic month is fixed at approximately 0.925196. That difference accumulates with each lunar tropical month, i.e., 1.850392 after two lunar tropical months, 5.551176 after six, and so on. According to lunar data from the Astronomical Almanac, an Earth equinoctial year, in which Earth has completed 359° 59' 10" of its 360° orbit around the Sun, consists of approximately 13.368267 lunar tropical months and approximately 12.368267 synodic months. The difference between the two figures, according to the lunar data, is equal to one.[27] Because of Earth's orbit around the Sun, however, the difference between the two figures should only reach one when Earth has completed a full 360° orbit of the Sun. It is a geometric impossibility for the difference between the two figures to be equal to one in an Earth equinoctial year unless an Earth equinoctial year actually constitutes a 360° orbit around the Sun.[28]

Likewise, if Earth and the Moon are permitted to continue their respective orbits for an additional twenty minutes – so as to allow Earth to sweep out the remaining 50 arseconds that extends an Earth equinoctial year into an Earth sidereal year – the lunar data show that there have been approximately 13.368785 lunar tropical months and approximately 12.368746 synodic months. The difference of 1.00003899 implies that Earth has traveled a non-trivial amount more than 360° around the Sun, which again is a geometric impossibility unless either the VLBI data are incorrect – which is highly unlikely – or an Earth equinoctial year is actually 360° instead of 359° 59' 10" (and therefore an Earth sidereal year is greater than 360°).

What does this all mean? Is the lunisolar theory of precession, and indeed the very idea itself that Earth undergoes axial precession, open to a charge of what Stephen Weinberg calls fine-

[27] The lunar data actually show a difference of 1.00000018.

[28] Cruttenden, p. 8.

tuning? "We criticize a proposed theory as fine-tuning when its features are adjusted to make some things equal, without any understanding of why they should be equal. The approach of fine-tuning in a scientific theory is like a cry of distress from nature, complaining that something needs to be better explained."[29] Did Copernicus posit a phantom description to solve a problem that, to this very day, is not yet fully understood? While it is true that VLBI measurements are made only with respect to objects well outside the Solar System, and not with respect to objects inside the Solar System, the precessional phenomenon it measures is very well measured by it and is very real, which means that something is moving. Yet considering (i) the seemingly insoluble challenge of formulating a precession equation that yields accurate, or even reasonable, predictions over long periods of time, (ii) the gravity-defying observation that precession is accelerating while the Moon is receding from Earth and Earth is receding from the Sun, and (iii) the lunar data suggesting that Earth appear to be precessing only with respect to objects outside the Solar System, one cannot help but wonder whether a form of fine-tuning may be plaguing the lunisolar theory of precession.

What then would it mean if Earth were not undergoing axial precession as Copernicus thought? There are alternative proposals, all beyond the scope of this paper to explore in detail, that suggest that another frame of reference might be at work. One such proposal is that Earth is indeed not undergoing axial precession. Instead, the precessional phenomenon is the result of the entire Solar System curving through space, clockwise, in a Keplerian orbit around another local mass, even as the Solar System and that other mass are also in orbit around the galactic center.[30] The addition of this purportedly missing observational reference frame converts the Copernican reference frame into, essentially, a reference frame within a reference frame.

This alternative model makes several intriguing claims. First it posits, in Keplerian terms, an explanation for the observed acceleration in the current rate of precession. That is, while the Solar System's current velocity around the barycenter of its obit with the other mass is such as to be consistent with a 50 arcsecond per year apparent motion of the precessional phenomenon, the Solar System is also moving towards that barycenter and so, consistent with Kepler's Second Law, both the Solar System's velocity and the apparent rate of precession are slowly increasing. By extension, the alternative model should also predict that the precessional rate will continue to accelerate before eventually decelerating after reaching periapsis.[31] Likewise this model renders essentially irrelevant, for purposes of precession, the observed receding of the Moon from Earth, and Earth from the Sun. If the precessional phenomenon is the result of the Solar System in orbit around another local mass, and not the result of Earth's axial precession, then the apparent inconsistency between the accelerating rate of the precessional phenomenon and the lunisolar demands of universal gravitation becomes moot. Finally, this alternative model also resolves the lunar data-based paradox that the precessional phenomenon appears to be observable only with respect to objects outside the Solar System. If Earth is not undergoing axial precession, then Earth must be completing a full 360° orbit, not merely 359° 59' 10", around the Sun in an equinoctial

[29] Weinberg, p. 82.

[30] See Cruttenden.

[31] Periapsis is the point in the path of an orbiting body at which it is nearest to the body that it orbits.

year. The missing 50 arcseconds per year are instead an observational phantom within the Solar System – the result of failing to take into account the missing observational reference frame of the Solar System's orbit around another local mass – but is therefore very real outside the Solar System.

Though this alternative model explains these and other discrepancies between the Copernican model's description of the precessional phenomenon, and the theoretical and observational underpinnings to which that explanation is ultimately accountable, the alternative model also raises questions about itself. Where is this other local mass, and how is it that it has not yet been identified?[32] What is to be done with lunisolar theory and how, for example, does lunisolar theory continue to provide a theoretical basis for other harmonics like nutation, but not axial precession? Finally, even if some axial precession is still permitted, how are the predictions of lunisolar theory to be radically scaled back while at the same time continuing to explain, for example, the ocean tides?

Perhaps the Copernican model of precession is not yet ready for a Kuhnian paradigm shift, but the problematic evidence continues to mount and will eventually require either that the current model and its theoretical foundations come to explain much more fully the observed inconsistencies, or that the model be superseded by a new model than explains both what the current model does, and what it does not, explain.

***

BIBLIOGRAPHY AND REFERENCES

Blair, Ann. "Tycho Brahe's Critique of Copernicus and the Copernican System." *Journal of the History of Ideas* 51, no. 3 (1990): 355–77.

Capitaine, Wallace and Chapront. "Expressions for IAU 2000 precessional quantities." *Astronomy & Astrophysics* 412 (2003): pp. 567-586.

Close, Maxwell H. "Hipparchus and the Precession of the Equinoxes." *Proceedings of the Royal Irish Academy* 6 (1900): pp. 450–56.

Cruttenden, Walter. "Earth Orientation: Does Solar System Motion Matter?" Unpublished manuscript.

Dobson, Geoffrey J. "On Lemmas 1 and 2 to Proposition 39 of Book 3 of Newton's Principia." *Archive for History of Exact Sciences* 55, no. 4 (2001): pp. 345-363.

Gattano, Lambert and Bizouard. Observation of the Earth's Nutation by the VLBI: How Accurate is the Geophysical Signal. *Journal of Geodesy* (2016): pp. 1-13.

[32] See Mattese and Whitmore on one possibility, or at least one type of possibility.

Hilton, Capitaine, Chapront, Ferrándiz, Fienga, Fukushima, Getino, Mathews, Simon, Soffel, Vondrak, Wallace and Williams. "Report of the International Astronomical Union Division I Working Group on Precession and the Ecliptic." *Celestial Mechanics and Dynamical Astronomy* 94(3) (2006): pp. 351-367.

International Astronomical Union. *IAU 2006 Resolution B1 of the XXVIth General Assembly* of the *International Astronomical Union* (2006).

International Astronomical Union. *IAU 2011 Resolution B2 of the XXXIst General Assembly of the International Astronomical Union* (2011).

Livio, Mario. "Did Galileo Truly Say, 'And Yet It Moves'? A Modern Detective Story." *Scientific American* (2000) (available at https://blogs.scientificamerican.com/observations/did-galileo-truly-say-and-yet-it-moves-a-modern-detective-story/).

Malkin, Z.M. "The Influence of Galactic Aberration on Precession Parameters Determined from VLBI Observations." *Astronomy Reports* 55, no. 9 (2011): pp. 810–815.

Mattese, John and Whitmore, Daniel. "Persistent Evidence of a Jovian Mass Solar Companion in the Oort Cloud." *Icarus* 211 no. 2 (2011): pp. 926-938.

Newcomb, Simon. "On the Value of the Precessional Constant." *Astronomical Journal* No. 359 (1895): 185-188.

Newton, Isaac. *The Principia: The Authoritative Translation : Mathematical Principles of Natural Philosophy* (University of California Press, 2016).

Robinson, Abram. "On the Precession of the Equinoxes." *Philosophical Transactions of the Royal Society of London* 97 (1807): pp. 57-82.

Sandeman, William. *The Path of the Sun* (Sherratt & Hughes, 1900).

Weinberg, Stephen. *To Explain the World – The Discovery of Modern Science* (Penguin Books, 2016).